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TRANSIENT MOTION OF A FLOATING BODY HAVING A RECTANGULAR SHAPE

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The two-dimensional transient problem of a floating body having a rectangular shape in a fluid layer of finite depth is considered. Vertical displacements of the body are specified. The problem is studied within the framework of the linear theory of potential ideal incompressible flow. The fluid flow equations reduce to an infinite system of Volterra integral equations of the second kind by the method of decomposition of the flow region. The system obtained is studied and solved numerically by the reduction method. A method of solving the problem for the flow velocity potential is proposed. The distribution of the hydrodynamic pressure and force acting on the body is determined.

Introduction. The behavior of a floating body is a classical problem of ship hydrodynamics. At present, it has become important in connection with the design of floating airfields and space rocket launching pads. Work on these buildings is reasonable only when their displacements are small.

Smallness of the departure of a floating body from the equilibrium position makes it possible to study body motion within the framework of the linear theory of potential ideal incompressible flow. In linear theory, the boundary conditions are linearized and extended to the equilibrium position of fluid free boundary and surface of the floating body. In particular, it is assumed that in calculations of the hydrodynamic forces acting on the body, changes of the wetted part can be ignored. Despite of the adopted simplifications, the problem remains rather complex because the shape of the immersed part of the body is not necessarily canonical; the hydrodynamic forces depend on fluid depth and the history of the process; the fluid flow is described by the solution of the boundary-value problem for the velocity potential in a region of complex shape with mixed conditions on its boundary; within the framework of linear theory, singularities of the velocity field can arise on the lines of change of the type of boundary condition and at the angular points of the body surface.

The problem can be solved only by numerical methods, but in constructing an adequate algorithm, one should allow for features of fluid flow and body motion.

There are two methods for constructing a solution of the problem considered. The first method (frequency-domain method) [1] uses the Fourier transform over time, and a new parameter — frequency — appears in this case. The final solution is obtained using the inverse Fourier transform. The second method (time-domain method) is used to solve the transient problem itself. A drawback of the second method is that to determine unknown quantities, one need to know the entire history of the process. This can increase the computation time considerably. The error of the initial stage of calculation increases during the entire calculation process. At the same time, when using the first method, it is necessary to solve a large number of problems for various values of frequency.

Interest in the development of the second method has recently increased. The Green function is constructed on the basis of the fundamental solution of the Laplace equation in the flow region, and the problem reduces to integral equations on the boundary of the region. The work of Cummins [2] played an important role in the development of this method. Johansson [3] performed a comparative analysis of the method proposed in [2] and the method of solving the problem in the range of frequencies using as an example a vertical circular cylinder floating in a fluid of finite depth. A large number of papers have been devoted to the solution of the problem of a floating body using the method of boundary integral equations. Watanabe [4] studied the interaction of a fluid and a floating thin elastic plate acted upon by a point load (model of floating airfield landing) works. Duan and Dai [5] determined

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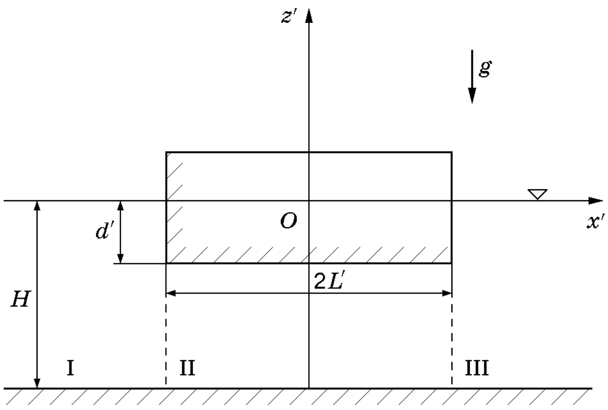

Fig. 1

the velocity potential for a floating body with oblique side walls. Xia and Wang [6] proposed a variational method for solving the problem of a floating prolate body.

In the method of boundary integral equations (boundary-element method) the infinite region is replaced by a bounded region. To increase the accuracy of results, besides decreasing the grid cell, it is necessary to extend the computation domain, which increases the computation time considerably. In the present paper, we propose a method for solving the problem in a two-dimensional formulation.

Formulation of the Problem. The transient problem of behavior of a floating body in a fluid layer of depth H is considered. The body has the shape of a prolate rectangular parallelepiped. The bottom is considered even, and the draught of the body is d'. The fluid is ponderable, ideal, and incompressible, and its flow is potential.

In the case of a prolate body (the transverse dimensions of the body are much larger than the longitudinal dimensions) it is possible to use the theory of flat sections [1]. The coordinate system is located so that its origin is at the level of the unperturbed fluid surface, the Oz' axis passes through the center of gravity of the body, and the Oy' axis is directed along the body (Fig. 1). Then, in each section $y' = \text{const}$, the velocity potential Φ' can be treated as a function of the variables x', z', and t'.

Let a rectangular body having length $2L'$ and draught d' perform specified vertical displacements $z' = \xi'(t')$ of small amplitude, and $\xi'(0) = \dot{\xi}'(0) = 0$ (the dot denotes the derivative with respect to time). We introduce nondimensional variables as follows: $x' = xH$, $z' = zH$, $L' = LH$, $d' = dH$, $\xi'(t') = \xi(t)H$, $t' = t\sqrt{H/g}$, $\Phi' = H\sqrt{gH}\Phi(x,z,t)$, and $p' = p\rho g H$. Here $p(x,z,t)$ is the hydrodynamic pressure, g is the free-fall acceleration, and Φ is the flow velocity potential. Primed variables are dimensional, and unprimed variables are nondimensional.

Within the framework of the linear theory of potential ideal incompressible flow, the equations of motion, the boundary conditions, and the initial data in the nondimensional variables are written as

$$\Phi_{xx} + \Phi_{zz} = 0, \qquad (x,z) \in \Omega; \tag{1}$$

$$\Phi_x = 0 \quad \text{for} \quad x = \pm L,\ z \in [-d, 0]; \tag{2}$$

$$\Phi_z = 0 \quad \text{for} \quad z = -1; \tag{3}$$

$$\Phi_z = \dot{\xi}(t) \quad \text{for} \quad z = -d,\ |x| < L; \tag{4}$$

$$\Phi_{tt} + \Phi_z = 0 \quad \text{for} \quad z = 0,\ |x| > L; \tag{5}$$

$$\Phi \to 0 \quad \text{as} \quad |\boldsymbol{x}| \to \infty; \tag{6}$$

$$\Phi = \Phi_t = 0 \quad \text{for} \quad t = 0. \tag{7}$$

Here $\Omega = (((-\infty, -L) \cup (L, \infty)) \times [-1, 0]) \cup ([-L, L] \times [-1, -d])$. Equations (1)–(7) form the initial boundary-value problem for the flow velocity potential.

Method of Decomposition. Volterra System. The solution of problem (1)–(7) is constructed by the method of decomposition. The flow region is divided into three parts (Fig. 1): the left exterior region I ($x < -L$), the interior region II ($-L < x < L$), and the right exterior region III ($x > L$).

The problem is symmetric about the Oz axis, and the line $x = 0$ is a streamline:

$$\Phi_x = 0 \qquad \text{for} \quad x = 0. \tag{8}$$

Therefore, it suffices to solve problem (1)–(7) for two regions: $(x, z) \in [0, L] \times [-1, -d]$ (interior problem) and $(x, z) \in [L, \infty) \times [-1, 0]$ (exterior problem). On the line of contact of the two regions, the continuity condition for the horizontal velocity component and the velocity potential should be satisfied. Let, on the contact line, the distribution of the horizontal velocity is described by a certain function $f(z, t)$:

$$\Phi_x = f(z, t) \qquad \text{for} \quad x = L, \ z \in [-1, -d]. \tag{9}$$

Equations (1), (3), (4), (8), and (9) form the interior boundary-value problem for the velocity potential Φ with the Neumann boundary conditions, Eqs. (1)–(3), (5)–(7), and (9) form the exterior problem. The solution of the Neumann problem (1), (3), (4), (8), (9) exists if the following condition is satisfied:

$$\int\limits_{-1}^{-d} f(\eta, t)\, d\eta + \dot{\xi}(t) L = 0. \tag{10}$$

Solving the interior problem by the Fourier method, we find the velocity potential $\Phi_{\rm I}(x, z, t)$ in the form of the series

$$\Phi_{\rm I}(x, z, t) = \frac{\dot{\xi}(t)}{2d_1}[(z+1)^2 - x^2] + \alpha_0(t) + \sum_{n=1}^{\infty} \frac{\cosh\,(\lambda_n x)}{\lambda_n \sinh\,(\lambda_n L)}\, f_n(t) \cos\,(\lambda_n(z+1)), \tag{11}$$

where $\lambda_n = \pi n/d_1$, $d_1 = 1 - d$, and f_n are the coefficients of the Fourier expansion of the horizontal velocity $f(z, t)$ in the complete orthogonal system of functions $\cos\,(\lambda_n(z+1))$, i.e.,

$$f(z, t) = \sum_{k=0}^{\infty} f_k(t) \cos\,(\lambda_k(z+1)). \tag{12}$$

From the condition (10) we obtain $f_0(t) = -(L/d_1)\dot{\xi}(t)$.

We change the variable $x = \tilde{x} + L$. In the variables $\tilde{x}$ and z the right exterior region moves to the left by L, and the equations do not vary.

For $\tilde{x} > 0$, the velocity potential $\Phi_{\rm II}(\tilde{x}, z, t)$ is sought as the sum of two terms φ_0 and φ, where φ_0 satisfies condition (9). Condition (5) is replaced by the following:

$$\varphi_0 = 0 \qquad \text{for} \quad \tilde{x} > 0, \ z = 0.$$

For φ, condition (9) is replaced by the homogeneous condition

$$\varphi_{\tilde{x}} = 0 \qquad \text{for} \quad \tilde{x} = 0, \tag{13}$$

and the free-surface condition becomes

$$\varphi_{tt} + \varphi_z = -(\varphi_0)_z(\tilde{x}, z, t) \qquad \text{for} \quad \tilde{x} > 0, \ z = 0.$$

Similarly to the interior problem, the solution for φ_0 is represented as the series

$$\varphi_0 = -\sum_{n=0}^{\infty} \frac{2}{\mu_n} \exp\,(-\mu_n \tilde{x}) \beta_n(t) \cos\,(\mu_n(z+1)),$$

where $\mu_n = (\pi/2)(2n+1)$ and

$$\beta_n(t) = \int\limits_{-1}^{-d} f(\eta, t) \cos\,(\mu_n(\eta+1))\, d\eta \qquad (n = 0, 1, 2, \ldots). \tag{14}$$

To find the function $\varphi(\tilde{x}, z, t)$, we continue it symmetrically about the $\tilde{x} = 0$ axis. Then, the nonpenetration condition (13) on this line is satisfied automatically, and the free-surface condition becomes

$$\varphi_{tt} + \varphi_z = -(\varphi_0)_z(|\tilde{x}|, z, t) \qquad \text{for} \quad z = 0. \tag{15}$$

By the direct Fourier transform, system (1), (3), (6), (15) is reduced to the system

$$\varphi^F_{zz} - \zeta^2 \varphi^F = 0,$$

$$\varphi_{tt}^F + \varphi_z^F = h^F(\zeta, z, t) \qquad \text{for} \quad z = 0,$$

$$\varphi_z^F = 0 \qquad \text{for} \quad z = -1,$$

where

$$h^F = -\int_{-\infty}^{\infty} (\varphi_0)_z(|\tilde{x}|, z, t) \exp(-i\zeta\tilde{x})\, d\tilde{x} = -4\sum_{n=0}^{\infty} (-1)^n \frac{\mu_n}{\mu_n^2 + \zeta^2}\, \beta_n(t).$$

Solving this system with zero initial data and using the inverse Fourier transform, we obtain

$$\varphi = -\frac{2}{\pi}\int_{-\infty}^{\infty}\int_{0}^{t} \frac{\sin(\omega(t-\tau))}{\omega}\, \frac{\cosh(\zeta(z+1))}{\cosh\zeta} \Big\{\sum_{n=0}^{\infty} (-1)^n \frac{\mu_n \beta_n(\tau)}{\mu_n^2 + \zeta^2}\Big\} \exp(i\zeta\tilde{x})\, d\tau\, d\zeta,$$

where $\omega^2 = \zeta \tanh \zeta$. Using (14) and the table of series from [7], it is possible to show that the expression in angular brackets is equal to

$$\frac{1}{2}\int_{-1}^{-d} f(\eta, \tau)\, \frac{\cosh(\zeta(\eta+1))}{\cosh\zeta}\, d\eta.$$

We finally have

$$\Phi_{\text{II}} = -\sum_{n=0}^{\infty} \frac{2}{\mu_n} \exp(-\mu_n\tilde{x}) \cos(\mu_n(z+1))\beta_n(t)$$

$$-\frac{1}{\pi}\int_{0}^{t}\int_{-\infty}^{\infty}\int_{-1}^{-d} f(\eta,\tau)\, \frac{\cosh(\zeta(\eta+1))}{\cosh\zeta}\, d\eta \frac{\sin(\omega(t-\tau))}{\omega}\, \frac{\cosh(\zeta(z+1))}{\cosh\zeta} \exp(i\zeta\tilde{x})\, d\zeta\, d\tau. \tag{16}$$

On the line $x = L$, the velocity potential varies continuously, i.e., $\Phi_{\text{I}} = \Phi_{\text{II}}$. Equating expressions (11) and (16) on the line $x = L$, we obtain the following equation for the function $f(z,t)$:

$$\frac{\dot{\xi}(t)}{2d_1}[(z+1)^2 - L^2] + \alpha_0(t) + \sum_{n=1}^{\infty} \frac{f_n(t)}{\lambda_n \tanh(\lambda_n L)} \cos(\lambda_n(z+1)) = -2\sum_{n=0}^{\infty} \frac{1}{\mu_n} \cos(\mu_n(z+1))\beta_n(t)$$

$$-\frac{1}{\pi}\int_{0}^{t}\int_{-\infty}^{\infty}\int_{-1}^{-d} f(\eta,\tau) \frac{\cosh(\zeta(\eta+1))}{\cosh\zeta}\, d\eta \frac{\sin(\omega(t-\tau))}{\omega}\, \frac{\cosh(\zeta(z+1))}{\cosh\zeta}\, d\zeta\, d\tau. \tag{17}$$

Substituting series (12) for the function $f(z,t)$ into (14) and (17), we obtain

$$\beta_n(t) = \sum_{k=0}^{\infty} f_k T_{nk}, \qquad \int_{-1}^{-d} f(\eta, t) \frac{\cosh(\zeta(\eta+1))}{\cosh\zeta}\, d\eta = \sum_{k=0}^{\infty} f_k(t) D_k(\zeta),$$

where

$$\begin{gathered} T_{nk} = (-1)^k \frac{2}{\pi} \sin\Big(\pi\, \frac{(2n+1)d_1}{2}\Big) \frac{2n+1}{(2n+1)^2 - (2k/d_1)^2}, \\ D_k(\zeta) = (-1)^k \frac{\zeta}{\zeta^2 + (\pi k/d_1)^2}\, \frac{\sinh(\zeta d_1)}{\cosh\zeta}. \end{gathered} \tag{18}$$

The system $\{\cos(\lambda_n(z+1))\}_{n=0}^{\infty}$ is complete and orthogonal. Multiplying Eq. (17) by $\cos(\lambda_m(z+1))$ and integrating over z from -1 to $-d$, we obtain an infinite system of integral equations, which can be written in matrix form

$$\begin{gathered} A\boldsymbol{f}(t) + \int_{0}^{t} S(t-\tau)\boldsymbol{f}(\tau)\, d\tau = \boldsymbol{F}(t), \\ \alpha_0(t) = -\frac{1}{1-d}\left(\boldsymbol{P}\boldsymbol{f}(t) + \int_{0}^{t} \boldsymbol{S}_0(t-\tau)\boldsymbol{f}(\tau)\, d\tau\right) + F_0(t). \end{gathered} \tag{19}$$

Here $A = \{A_{km}\}_{k,m=1}^{\infty}$ and $S = \{S_{km}\}_{k,m=1}^{\infty}$ are symmetric matrices with the elements

$$A_{km} = \frac{d_1}{2} \frac{1}{\lambda_m \tanh(\lambda_m L)} \delta_{km} + 2 \sum_{n=0}^{\infty} \frac{T_{nk} T_{nm}}{\mu_n}, \qquad S_{km}(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\sin(\omega t)}{\omega} D_k(\zeta) D_m(\zeta) \, d\zeta,$$

$$\boldsymbol{P} = \Big\{ 2 \sum_{n=0}^{\infty} \frac{T_{nk} T_{n0}}{\mu_n} \Big\}_{k=1}^{\infty}, \qquad F_0(t) = \dot{\xi}(t) \Big(\frac{L P_{00}}{d_1^2} - \frac{d_1^2 - 3L^2}{6 d_1} \Big) + \frac{L}{d_1^2} \int_0^t \dot{\xi}(\tau) S_{00}(t - \tau) \, d\tau,$$

$$\boldsymbol{f} = \{f_k\}_{k=1}^{\infty}, \qquad \boldsymbol{S}_0 = \{S_{k0}\}_{k=1}^{\infty}, \qquad \boldsymbol{F} = \{F_m\}_{m=1}^{\infty}$$

(δ_{km} is the Kronecker delta). The components of the vector $\boldsymbol{F}(t)$ are calculated from the formulas

$$F_m(t) = \dot{\xi}(t) \Big(\frac{L}{d_1} P_{0m} + (-1)^{m+1} \Big(\frac{d_1}{\pi m} \Big)^2 \Big) + \frac{L}{d_1} \int_0^t \dot{\xi}(\tau) S_{0m}(t - \tau) \, d\tau.$$

Thus we obtain an infinite system of Volterra integral equations of the second kind for determining the coefficients of the Fourier expansion of the horizontal velocity on the line of contact of the interior and exterior regions.

Analysis of the System. We note that in expression (18) for T_{nk}, the denominator contains a difference that vanishes for a certain value of d (we call it the critical draught d_*). Nevertheless, by virtue of the limits

$$\lim_{d \to d_*(k)} \frac{\sin^2((\pi/2) d_1 (2n+1))}{(2n+1)^2 - (2k/d_1)^2} = 0 \qquad (k \neq m),$$

$$\lim_{d \to d_*(k)} \frac{\sin^2((\pi/2) d_1 (2n+1))}{\Big[(2n+1)^2 - (2k/d_1)^2\Big]^2} = \frac{(\pi k)^2}{4(2n+1)^4} \qquad (k = m),$$

the terms of the series entering into $S_{km}(t)$ have no singularities.

The kernels of system (19) are integrals over the entire numerical axis. By virtue of the evenness of the integrand functions, these integrals can be replaced by double integrals only on the positive semiaxis.

We note some properties of the functions $S_{km}(t)$. First, with increase in one the subscripts, the functions $S_{km}(t)$ decay, and beginning with a certain M ($|S_{km}| \leqslant \varepsilon$ at $k, m > M$), we can set $S_{km} \equiv 0$.

Second, $S_{km}(t)$ are symmetric about their subscripts. We introduce the functions

$$S_k(t) = \frac{2}{\pi} \int_0^{\infty} \frac{\sin(\omega t)}{\omega} \frac{1}{\zeta^2 + (\pi k/d_1)^2} \frac{\sinh^2(\zeta d_1)}{\cosh^2 \zeta} \, d\zeta,$$

in terms of which the elements of the matrix S_{km} at $k \neq m$ are expressed by the formula

$$S_{km}(t) = \frac{(-1)^{k+m}}{k^2 - m^2} \Big(k^2 S_k(t) - m^2 S_m(t) \Big).$$

From the last formula it follows that we do not need to calculate $(M^2 + M)/2$ functions $S_{km}(t)$ (for $k \leqslant m$ by virtue of symmetry of the matrix) but it suffices to calculate only $M + 1$ functions S_{kk} ($k = 0, 1, \ldots, M$) and M functions S_k ($k = 1, 2, \ldots, M$), i.e., $2M + 1$ functions.

Third, as $t \to \infty$, the functions $S_{km}(t)$ behave as $O(t^{-4})$. Because $S_{km}(t)$ are expressed linearly in terms of $S_k(t)$, it suffices to show the validity of this statement for the functions S_k. We denote

$$G_k(\zeta) = \frac{2}{\pi} \frac{1}{\zeta^2 + (\pi k/d_1)^2} \frac{\sinh^2(\zeta d_1)}{\cosh^2 \zeta}$$

and change the variable $\omega^2(\zeta) = \zeta \tanh \zeta$. In the neighborhood of zero, ω behaves as ζ, and at infinity, it behaves as $\sqrt{\zeta}$. Let $g_k(\omega) = G_k(\zeta(\omega))$, and, hence,

$$S_k(t) = \int_0^{\infty} \frac{\sin(\omega t)}{\omega} G_k(\zeta) \, d\zeta = \int_0^{\infty} \frac{g_k(\omega)}{\omega \omega'} \sin(\omega t) \, d\omega \tag{20}$$

(prime denotes differentiation with respect to ζ).

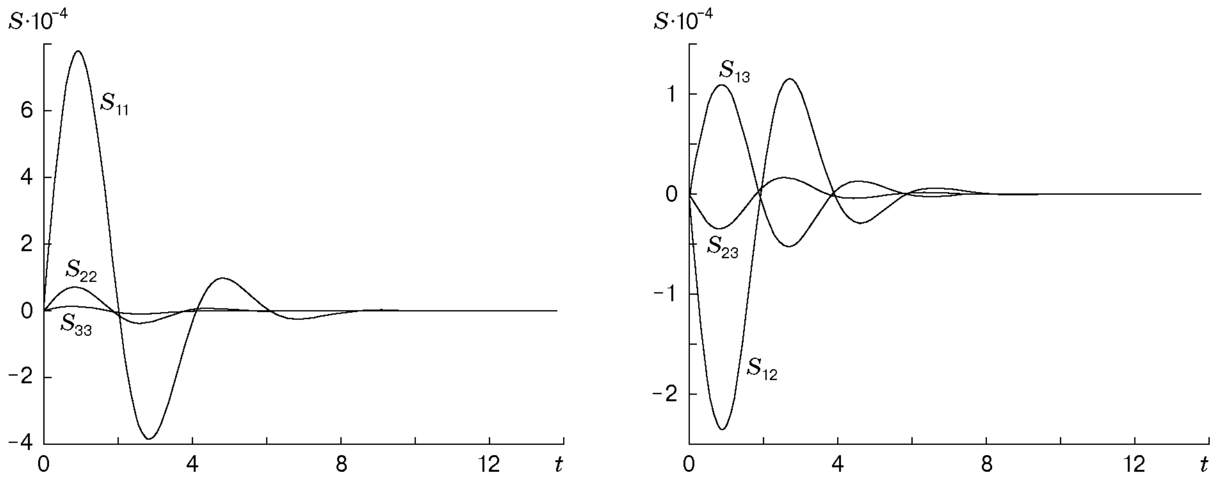

Fig. 2

We denote $h(\omega) \equiv g_k(\omega)/(\omega\omega')$ and consider two extreme cases. In the neighborhood of zero, we have $h(\omega) \approx 2d_1^4\omega/(\pi^3 k^2)$, i.e., $h(\omega) = O(\omega)$ and $h'(0) = 2d_1^4/(\pi^3 k^2) < \infty$. In the neighborhood of the infinite point $h(\omega) \approx (4/\pi)\omega^{-4}$, i.e., $h(\omega) = O(\omega^{-4})$ and $h^{(N)}(\omega) = O(\omega^{-(N+4)})$ at $N > 1$.

Integrating (20) by parts and taking into account the behavior of the function $h(\omega)$ and its derivatives, we show that

$$S_k(t) = \frac{1}{t^4} \int\limits_0^\infty \sin(\omega t) h^{(4)}(\omega)\, d\omega = O(t^{-4}) \qquad \text{as} \quad t \to \infty.$$

Beginning from a certain time T such that $|S_k(t)| \leqslant \varepsilon$ for $t \geqslant T$, we can set $S_k(t) \equiv 0$ for $t \geqslant T$.

Calculations for the matrix $S(t)$ for $0 \leqslant t \leqslant 20$ (step equal to 0.1) and draught $d = 0.4$ show that for $t > 10$, we can set $S(t) \equiv 0$ (Fig. 2). From Fig. 2 it can be seen that the amplitude decreases with increase in matrix element number.

Numerical Solution. To determine the vector $\boldsymbol{f}$, we solve the Volterra system (19) by the reduction method: $\boldsymbol{f} = \{f_k(t)\}$ for $k = 1, 2, \ldots, M$ and $f_k \equiv 0$ for $k > M$. The integrals are approximated by the trapezoid rule. Taking into account that at the initial time, $f(z, 0) = 0$, we obtain the computational formulas

$$\boldsymbol{f}(0) = 0, \qquad \boldsymbol{f}(N\Delta\tau) = A^{-1}\boldsymbol{F}(N\Delta\tau) - A^{-1}\sum_{n=1}^{N-1} S((N-n)\Delta\tau)\boldsymbol{f}(n\Delta\tau),$$

$$\alpha_0(N\Delta\tau) = -\frac{1}{1-d}\Big(\boldsymbol{P}\boldsymbol{f}(N\Delta\tau) + \sum_{n=1}^{N-1} \mathbf{S}_0((N-n)\Delta\tau)\boldsymbol{f}(n\Delta\tau)\Delta\tau\Big) + F_0(N\Delta\tau).$$

From analysis of numerical calculations for a body with $d = 0.4$ moving under the law $\xi(t) = (1 - \exp(-t))\sin t$, it follows that after a certain time, the solution becomes almost harmonic in time with period 2π.

In the neighborhood of the angular point of the body $(L, -d)$, the velocity potential Φ behaves as $O(r^{2/3})$, where $r^2 = (x-L)^2 + (z+d)^2$. The function $f(z, t)$ is a horizontal velocity on the line $x = L$, and, hence, it has a singularity of the form $(z+d)^{-1/3}$ as $z \to -d - 0$. It is easy to show that with increase in number, the coefficients $f_n(t)$ should decay as $O(n^{-2/3})$. For large values of n, the quantities $f_n(0.2)$ (see Fig. 3) behave as $Cn^{-2/3}$, where $C = 0.265621$. In Fig. 3, the points show the values of $f_n(0.2)$ for $n = 1, 2, \ldots, 50$, and the curves are the functions $\pm Cn^{-2/3}$. It is obvious that the calculation results are in good agreement with theoretical data.

Hydrodynamic Force and Pressure. At the bottom of the body, the pressure is calculated from the formula $p_b(x, t) = -\xi(t) - (\Phi_{\mathrm{I}})_t(x, -d, t)$. Here the first term is the hydrostatic pressure component, and the second is the dynamic component. The expression for pressure does not contain a constant part that corresponds to the buoyancy force because this part is compensated for by gravity. From the expression for the dynamic pressure component

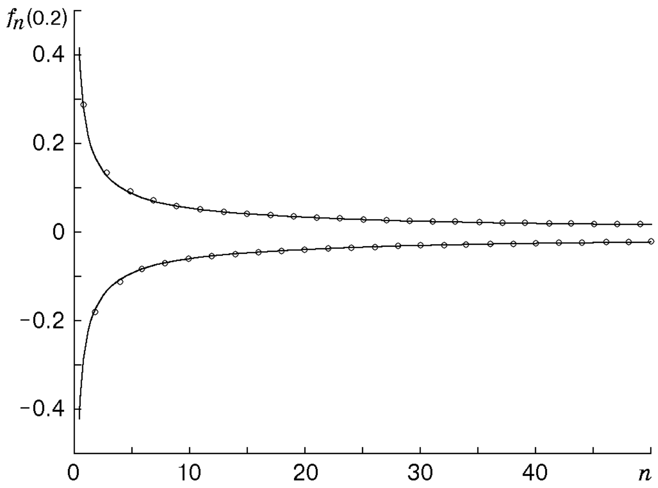

Fig. 3

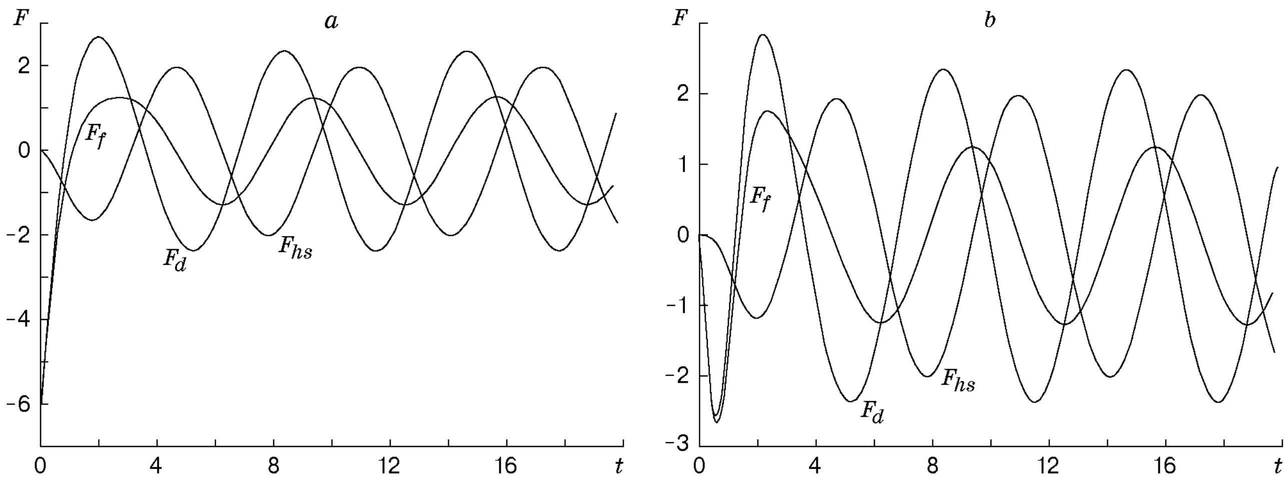

Fig. 4

$$(\Phi_{\mathrm{I}})_t(x,-d,t) = \frac{\ddot{\xi}(t)}{2d_1}(d_1^2 - x^2) + \dot{\alpha_0}(t) + \sum_{n=1}^{M} \frac{(-1)^n}{\lambda_n}\,\frac{\cosh\,(\lambda_n x)}{\sinh\,(\lambda_n L)}\,\dot{f}_n(t)$$

it follows that the derivatives of the functions $f_n(t)$ need to be known. These derivatives are approximated by central differences, except for the derivative at zero, which is approximated by the forward difference. The force exerted on the body by the fluid is equal to the integral of pressure over the bottom of the body.

We performed numerical calculations of the force for two laws of body motion: $\xi(t) = (1 - \exp(-t))\sin t$ (Fig. 4a) and $\xi(t) = (1 - \exp(-t))^3 \sin t$ (Fig. 4b). Figure 4 shows time dependences of hydrodynamic forces [F_d is the dynamic force component, F_{hs} is the hydrostatic force component, and F_f is the total force (sum of the hydrostatic and dynamic components)]. We note that the difference is observed only at the initial stage, where the equations of motion have different smoothness and the force is proportional to the second derivative of the motion function.

Thus, the method described above is stable and can be used to describe the action of a fluid on a floating body.

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REFERENCES

1. J. Newman, *Marine Hydrodynamics*, MIT Press, Cambridge (1977).
2. W. E. Cummins, "The impulse response function and ship motions," *Schiffstechnik*, **9**, No. 47, 101–109 (1962).
3. M. Johansson, "Transient motions of large floating structures," Report, Ser. A14, Chalmers Univ. of Technol., Göteborg (1986).
4. E. Watanabe and T. Utsunomiya, "Transient response analysis of a VLFS at airplane landing," in: *Proc. of the Int. Workshop on Very Large Floating Structures* (Hayama, Kanagawa, Jpn., November 25–28, 1996), Ship Res. Inst.–Shonan Village Center, Hayama, Kanagawa (1996), pp. 243–247.
5. W. Duan and Y. Dai, "Hydrodynamic analysis of non-wallsided floating body by time-domain method," *Proc. OMAE*, **1A**, 251–257 (1997).
6. J. Xia and Z. Wang, "Time-domain hydroelasticity theory of ship responding to waves," *J. Ship Res.*, **41**, No. 4, 286–300 (1997).
7. A. P. Prudnikov, Yu. A. Brychkov, and O. I. Marichev, *Integrals, Series, and Elementary Functions* [in Russian], Nauka, Moscow (1981).